
Three-dimensional model of the structure and evolution of coronal mass ejections (CME)

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**ABSTRACT**

We present a new three-dimensional magnetohydrodynamic (MHD) model which describes the evolution of coronal magnetic arcades in response to photospheric flows. The dynamics of the system is discussed and it is shown that the model reproduces a number of features characteristic of CME observations. In particular, we propose explanations for the three-part structure of CMEs observed at the solar limb and the formation and evolution of sigmoids and overlying arcades. The model includes the effects of finite resistivity in the system and morphological changes induced by reconnection are studied. Reconnection is found to prevent the formation of highly twisted magnetic structures if the magnitude of photospheric velocity is close to the observed values. In addition, we suggest a novel explanation for the splitting of the CME's core prominence material observed in some eruptions. The distinguishing features of the model are the novel numerical methods used to evolve the MHD system and the formulation of the boundary conditions. The stiffness of the resistive MHD equations constitutes a major difficulty for numerical simulations. Calculations in our model are performed using recently introduced exponential propagation techniques which allow efficient integration of the equations with time steps far exceeding the CFL bound that constrains explicit schemes.

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1. Introduction

Large explosions called coronal mass ejections (CME) inject billions of tons of magnetized plasma from the upper solar atmosphere into interplanetary space (Gosling 1997; Klimchuk 2000). The scale and frequency of these events (several times a day to a few a week) (St. Cyr O.C. et al. 2000) make CMEs one of the most important contributors to space weather (St. Cyr O.C. et al. 2000; Richardson et al. 2000; McAllister et al. 1996). Due to both observational limitations and intrinsic complexity, the three-dimensional structure of CMEs and their eruption mechanism are subjects of debate (Klimchuk 2000). CMEs are normally observed at the solar limb projected against the plane of sky or viewed against the solar disk as halo CMEs. The optically thin nature of the corona makes it difficult to discern the three-dimensional structure of a CME from the two-dimensional projection images obtained by observational instruments. Thus, modeling efforts are essential for both interpreting the observational data and understanding the nature of the coronal dynamics.

The models which address the question of the three-dimensional structure of eruptive coronal configurations and trace the evolution of such configurations analytically or through numerical simulations are generally divided into two groups (Klimchuk 2000). Some of the models advocate highly twisted structures called flux ropes, (e.g. Gibson & Low (2000); Forbes & Isenberg (1991); Wu & Guo (1997); Amari et al. (2000)), while others argue that the plasma configurations consist of sheared magnetic arcades, (Antiochos et al. (1999)). Since the MHD system which describes the evolution of the coronal plasma is quite complex and stiff, solution of these equations in three-dimensions is a challenging task. In fact, the majority of the existing models are based on 2.5 dimensional geometries (Forbes & Isenberg 1991; Wu & Guo 1997; Antiochos et al. 1999) and only a few recent investigations study the phenomenon in a fully three-dimensional setting (Gibson & Low 2000; Amari et al. 2000). Both of the latter references advocate that a flux rope is essential for explaining the structure and evolution of CMEs. By imposing a particular spatial and temporal profile of the boundary conditions Amari et al. (2000) create a highly twisted flux rope overlaid by arcade-like magnetic field lines and study the initiation mechanism for this configuration. Gibson & Low (2000) address the issues of the three-dimensional morphology of CMEs and successfully demonstrate that the spheroamak-type analytical MHD solutions they obtain can produce features that resemble some observational characteristics of the eruptions. The Gibson & Low (2000) model, however, does not address the questions of how such configurations form
initially and what their stability properties are. While flux rope geometry is useful for explaining some characteristics of CMEs, observational evidence supporting such structure is indirect and limited and the question of the existence of the flux ropes is still open.

In this paper we present a three-dimensional MHD model which investigates the effect of the photospheric velocity on the structure of the magnetic configurations. We show that the combination of converging and shearing flows at the photosphere can cause a formation of complex magnetic configurations that can account for the characteristic features seen in CME observations. In section 2 we formulate the model and describe the initial and boundary conditions and their influence on the solution of the MHD equations. In order to overcome difficulties associated with solving the system numerically we use recently introduced exponential propagation methods which allow us to efficiently integrate the MHD equations over hundreds of Alfven times. Section 3 gives a brief introduction to these methods and demonstrates how such numerical schemes can be constructed. The results of the simulations are presented in section 4 where we describe the dynamics exhibited by computed magnetic fields and relate it to the observations. Finally, we draw conclusions and sketch our future research plans in section 5.

2. Model description

We present a 3D numerical MHD model which describes how the large-scale coronal magnetic field dynamically responds to slow rotational motions of the photosphere. Since coronal $\beta$ (ratio of the hydrodynamic to the magnetic pressures) is very small ($\sim 10^{-3}$) we make the simplifying assumption of zero $\beta$ and uniform density profile (Zachary et al. 1994; Amari et al. 1996; Mikic et al. 1988). With these assumptions the MHD system is reduced to an induction equation and an equation of motion. The non-dimensional form of these equations is

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \frac{l}{S} \nabla^2 B,$$

$$\frac{\partial v}{\partial t} = -v \cdot \nabla v + (\nabla \times B) \times B + \frac{l}{R} \nabla^2 v,$$

where $B$ is the magnetic field vector, $v$ is the velocity vector. The two parameters $S$ and $R$ which appear are the Lundquist and the Reynolds numbers. These characterize the ratio of the resistive to Alfven times scales and the ratio of the viscous to Alfven time scales.
respectively. $S$ and $R$ are defined by

$$S = \frac{L^2/\eta}{\tau_A},$$

$$R = \frac{L^2/\nu}{\tau_A},$$

where $L$ is a characteristic spatial length scale, $\eta$ is the magnetic diffusivity, $\nu$ is the coefficient of kinematic viscosity and $\tau_A = L/v_A$ is the Alfven time defined using the Alfven velocity $v_A$. The majority of numerical MHD studies of coronal magnetic arcades set the viscosity time scale to be of order of $100\tau_A$ and the Lundquist number to be in a range $10^3 - 10^5$ (Mikic et al. 1988; Amari et al. 1996, 2000). This is done for numerical rather than physical considerations, because the real viscosity of the coronal plasma is extremely small and the Lundquist number is estimated to be $10^{12} - 10^{14}$. We adopted the conventional strategy of invoking a Lundquist number of $S = 4000$ and a rather low value $R$ in the numerical calculations and, in particular, ran simulations with $R = 100$ and $R = 10$. In both cases we found the structural changes discussed in Section 4 took place, however the rate at which the dynamical changes occur depended on the value chosen for $R$ (Tokman 2001).

In non-dimensionalized spatial variables our domain of integration is a volume $\{-1 \leq x \leq 1, -1 \leq y \leq 1, 0 \leq z \leq 2\}$; the base plane $z = 0$ corresponds to the photospheric surface. As an initial condition we use the potential magnetic field (Fig. 1a) generated by a magnetic dipole located below the base plane at $z_0 = -0.8$ and aligned parallel to the $y$ axis.

Our modeling of the driving mechanism of the system was inspired by laboratory simulations of prominence eruptions (Bellan & Hansen 1998) and is qualitatively similar to recent observations of rotating photospheric flows (Nightingale et al. 2000). By assuming that (i) the base plane dynamics is determined by the tangential electric field $E_i$ and (ii) the electric current $J_z$ at this plane has no tangential component, we calculate the base plane tangential velocity field $v_t = (v_x, v_y)$ from the MHD Ohm’s law as follows:

$$v_x = \frac{E_y}{B_z},$$

$$v_y = -\frac{E_x}{B_z}.$$  \hspace{1cm} (3)

The combination of the potential magnetic field and the tangential electric field generated by a bipolar electrostatic potential produces a velocity profile consisting of two co-rotating vortices. The following analytical function prescribes the tangential photospheric velocity which emulates such behavior

$$\psi(x, y, z = 0) = v_0 \frac{Cy^2}{(x^2 + y^2 + C^2)^{3/2}},$$  \hspace{1cm} (4)
- 5 -

\[ v_x(x, y, z = 0) = \frac{\partial \psi}{\partial y}, \]

\[ v_y(x, y, z = 0) = -\frac{\partial \psi}{\partial x}. \]

Here \( C = 0.5 \) and \( v_0 \) is also a constant chosen so that the maximum magnitude of the velocity is of the order of \( 0.01v_A \). Thus, in the simulations the spatial profile of the tangential velocity is maintained according to equations (6) and (7) as shown in Figure 1b while its magnitude is increased linearly from zero to a maximum value. The effects of varying the rate of change of the velocity magnitude and its maximum value in the simulations will be discussed in Section 4. While our reasoning in deriving the particular profile of a tangential velocity is different from previously published results, similar boundary conditions on \( \psi \) have been used in other numerical studies (Amari et al. 1996). In general, it can be argued that imposing the co-rotating vortices velocity profile at the base plane can be viewed as combining the converging and shearing flows considered in other models (Forbes & Isenberg 1991; Antiochos et al. 1999). We also impose the condition of no flow through the base plane.

We have considered two forms of boundary conditions on the magnetic field. Since the numerical model requires prescription of a tangential magnetic component at the boundary, we fixed the tangential component \( B_t \) at its initial value throughout the boundary of our Cartesian domain. The normal component \( B_n \) of the magnetic field was given in one of two different forms. In the first case the normal derivative of \( B_n \) at the boundary was calculated in such a way as to maintain the divergence free condition on \( \nabla \) up to and including the boundary, i.e. we imposed a Neumann condition

\[ \frac{\partial B_n}{\partial n} = -\nabla \cdot B_t. \]

In the second case a Dirichlet boundary condition was prescribed for the normal component and was fixed at its initial value \( B_n(x, y, z, t) = B_n(x, y, z, t = 0) \). The results of the simulations showed that the two kinds of boundary conditions affected the rates of the dynamics of the magnetic configurations, but the structural changes in the magnetic field were similar. In particular, the distinct features of the magnetic geometry which we describe in Section 4 were formed in both types of simulations. To check how such boundary conditions on the magnetic field affect the solution, we extended the side boundaries of the computational domain and compared the solutions of the standard domain \((2 \times 2 \times 2)\) to a much larger domain \((4 \times 4 \times 2)\). We found that the solutions matched well quantitatively and the geometry of the magnetic field lines in the standard domain matched the geometry of a corresponding volume inside the larger domain (Tokman 2001). Thus the formulation seems to give a good approximation to the realistic conditions of an open domain in the corona.
3. Numerical formulation and exponential propagation methods

The large Lundquist number $S$ introduces stiffness into the system (1)-(2). This poses a significant challenge in solving the equations numerically (Schnack et al. 1990; Amari et al. 1997). Stiffness corresponds to a wide separation of the fast and slow modes of the system. Thus, a severe CFL condition is imposed on the time step if the evolution is computed using explicit methods. In addition, the requirement of small size cells to obtain appropriate spatial resolution increases stiffness and makes the task of computing the solution over long time periods even more difficult. To overcome this problem one could use an implicit method in conjunction with Newton iteration. However, if a good preconditioner is not available, the convergence of an iterative method used to invert the stiff Jacobian matrix (as required for each single Newton iteration) will be very slow. We use recently introduced exponential propagation methods for nonlinear differential systems (Hochbruck & Lubich 1997; Hochbruck et al. 1998). These techniques allow us to obtain an accurate solution to the problem while using a time step larger than the CFL limit and speed up the convergence rate compared to an iterative method used to invert the stiff Jacobian. A detailed description of the mathematical theory behind these methods is beyond the scope of this paper. We instead briefly describe the main ideas behind the construction of an exponential propagation scheme and refer the interested reader to references (Hochbruck & Lubich 1997; Hochbruck et al. 1998; Gallopoulos & Saad 1992) for a more exhaustive treatment of the subject.

In order to apply this technique we first use the method of lines, i.e. we discretize spatial operators in (1)-(2) with fourth order finite-differencing to convert the system of partial differential equations into a large nonlinear system of ordinary differential equations (ODE). We define an $N = (6 \times n)$-dimensional solution vector to be

$$U = \begin{bmatrix} B^{i,j,k}_x \ B^{i,j,k}_y \ B^{i,j,k}_z \ v_x^{i,j,k} \ v_y^{i,j,k} \ v_z^{i,j,k} \end{bmatrix}^T,$$

$$1 \leq i \leq n_x, \quad 1 \leq j \leq n_y, \quad 1 \leq k \leq n_z,$$

where $n = n_x \times n_y \times n_z$ is the number of grid points in the computational domain. Applying the method of lines, we rewrite the system (1)-(2) as an initial value problem for $N$ nonlinear ODEs:

$$ \frac{\partial U(t)}{\partial t} = F(U(t)), \quad (9)$$

$$ U(t_0) = U_0, \quad (10)$$

where $F(U)$ is a discretized spatial MHD operator and $U_0$ is a vector which contains information about the initial and boundary conditions. Suppose now we want to compute a solution $U(t)$ of this system at time $t = t_0 + \Delta t$. If we define a Jacobian matrix of the system
at \( U = U_0 \) as
\[
A = \frac{\partial F}{\partial U}(U_0),
\]
and the nonlinear part of the system as
\[
R(U(t_0 + \Delta t)) = F(U(t_0 + \Delta t)) - F(U_0) - A(U(t_0 + \Delta t) - U_0),
\]
then we can write a formal integral form of the solution to the initial value problem (9)-(10)
\[
U(t_0 + \Delta t) = U_0 + \frac{e^{A\Delta t} - I}{A}F(U_0) + \int_0^{\Delta t} e^{A(t-\Delta t)}R(U(t_0 + s) - U_0)ds.
\]

The exponential propagation scheme is now constructed by approximating the second and the third terms on the right-hand-side of the equation (13). The second term is approximated using a Krylov subspace projection iteration, an idea first introduced for symmetric matrices by Nauts & Wyatt (1983) and then extended to non-symmetric case by Park & Light (1986) and Van der Vorst (1987). Finally, Hochbruck et al. (1998) utilized the framework of Runge-Kutta methods to approximate the nonlinear integral in the third term and construct an efficient and robust exponential propagation method with embedded automatic error control mechanism.

To integrate equations (1)-(2) in time we use the fourth order exponential Runge-Kutta method (equation (5.8) in Hochbruck et al. 1998). We have vectorized \( exp_4 \) software (Hochbruck et al. 1998) and embedded it as a time integrator in our MHD code. To benchmark our fourth order exponential propagation method, we compared it to a standard fourth order explicit Runge-Kutta scheme with the same error control mechanism (built by using a third order embedded method). The comparison was done for the standard 2.5 dimensional model presented in (Mikic et al. 1988). This comparison showed that in order to compute the solution for some specified accuracy, the time step in the exponential scheme can be 100 times larger than the maximum time step size allowed by the stability constraints for the explicit method. The overall integration time was reduced by a factor of 10-15 depending on the size of the grid and the final integration time used (Tokman 2001). While we plan to conduct a similar study for the three-dimensional model in the future, the 2.5D comparison provides an encouraging assessment of the capabilities of exponential propagation methods for MHD problems.
Two issues with regard to implementation of exponential propagation methods should be noted. First, since the complexity of exponential propagation methods comes mainly from the Krylov iteration, these methods have the same parallelization properties as Newton-Krylov implicit techniques. Thus, if computing the right-hand-side of (9) and calculating the product of a Jacobian matrix and a vector can be parallelized efficiently, then exponential propagation methods will be very effective on a parallel machine. Second, since the Krylov vectors need to be stored as the iteration proceeds, the memory requirements for the exponential propagation methods will be larger than for explicit schemes but comparable to Newton-Krylov implicit schemes based on iterative methods like GMRES. We stored a maximum of 15 Krylov vectors at each time step for the calculations presented in this paper. The runs with the largest grid size of $n = 84^3$ nodes were performed on a Cray SV1 vector supercomputer. The runs in a smaller domain ($n = 42^3$) and part of the convergence studies runs were completed on a Pentium 233 MHz workstation with 512 Mb of memory.

4. Three-dimensional structure and dynamics of magnetic configurations and related CME observations

The numerical simulations indicate that a simple prescribed rotational photospheric flow can cause the magnetic field to form complex structures with distinct subregions. When viewed from different angles these structures closely resemble features seen in CME observations. The resistive MHD approximation implies that the plasma is nearly frozen into the magnetic field. Thus, even though the brightness in the images from the white light observations corresponds to enhanced density regions rather than the magnetic field it is reasonable to expect a direct connection between the structure of the magnetic field lines and the white light profile seen in observations. While at present our model does not include evolution of the density profile it should be noted that equations (1)-(2) do not explicitly enforce fluid incompressibility. Thus, we can use the same argument as was previously presented in Mikic et al. (1988) and justified by other comparisons of simulations with uniform and non-uniform density profiles (Zachary et al. 1994; Amari et al. 1999) and suggest that the qualitative evolution of the field will be independent of the mass density profile. We can therefore expect that the same configurations will develop if density evolution is included in the model. The generic nature of the dynamics discussed below supports this hypothesis.

In the following subsections we describe the evolution of the computed magnetic field and relate it to particular features of CME observations. Our goal in this study was to invoke the simplest possible MHD model for the dynamical behavior and then study the evolution of the magnetic field. The model is constructed in a flexible way and could be
extended in the future to include more quantitative physics, but it is now premature to
make quantitative comparisons of exact rates of the physical processes occurring during a
CME with the numerical simulations. The calculations nevertheless indicate that the rate of
the dynamics depends on identifiable physical parameters. Since it is impossible to conduct
a full scale simulation with the exact coronal parameters (e.g. \( S \sim 10^{14} \)) given present
and foreseeable computer capabilities, it is important to study the qualitative parameter
dependencies in detail before trying to derive quantitative inferences.

4.1. Formation of sigmoid-arcade configurations and three-part structure of
magnetic field

About one third of CMEs observed in white light at the solar limb have a classical three-
part structure (Illing & Hundhausen 1983) (Fig. 2a). Such images typically display a bright
leading edge halo followed by a darker cavity and a bright feature located at the CME base.
When viewed from various angles the simulation configurations appear remarkably similar
to the observed images (Fig. 2b, c). In order to understand the geometry of such structures
let us discuss how subregions of distinct magnetic geometries develop.

The two distinct features of the computed configuration that bear the most striking
resemblance to the observed CME (Fig. 2a) are (i) the high-lying arcade (yellow and red
colored field lines in Figure 2b, c) which corresponds to the leading edge halo, and (ii) the
conic shape (orange-colored field lines in Figure 2b, c) that replicates the bright core of the
CME. How these structures form becomes evident when we study the behavior of different
magnetic field lines as their respective footpoints start rotating. Consider two concentric field
lines which initially lie in the \( x = 0 \) plane but have footpoints positioned differently with
respect to the centers of the rotating vortices (Fig. 3a). The footpoints of the yellow and
orange field lines are positioned respectively exterior and interior to the field line connecting
the vortex centers. Figure 3b shows the paths of the footpoints of these field lines as they
follow the rotational flow at the base plane. Since the footpoints of the orange field line
move away from each other in both the \( x \) and the \( y \) directions, this field line is stretched
and sheared by the flow to form a low-profile, sigmoidal shape (S-shape). In contrast, the
yellow field line is rotated nearly as a rigid body and becomes positioned perpendicular to
the S-shape. If we define a partition line to be the magnetic field line going from one vortex
center to the other (broken red line in Fig. 3a), then we can see that all field lines initially
positioned outside of the partition line will behave similarly to the yellow field line and all
field lines located inside the partition line will behave similarly to the orange field line. Thus,
two geometrically distinct regions of field lines form: (i) the field lines that form a sigmoid-
like structure and (ii) the field lines forming the overlaying arcade (Fig. 3c). The conic structure is a viewing artifact of the stacked S-shapes seen as a projection onto the xz plane. In order to clarify the structural features of the formed magnetic configurations we plot only representative field lines in Figure 3. Several large regions of geometrically distinct magnetic field lines form and can be identified this way. The characteristic features described above can be discerned even if the whole computational volume were filled with plotted field lines.

The structure of the numerically simulated CME reveals yet another feature which, to the best of our knowledge, has not been previously noted and which upon close examination is evident as a faint imprint in the observed CME in Figure 2a. This feature consists of a heart-shaped structure in the low density region of the cavity; in Figure 2d we have enhanced the brightness of this region so that the heart-shapes become more evident. The model shows that the shape of this feature is a projection effect from the two field lines positioned initially alongside of the partition line, i.e. in planes parallel to the x = 0 plane. Fig. 3d,e,f show the paths of the footpoints of these field lines and their structure as the evolution proceeds. Note that if viewed from above the field lines forming the heart shapes will look like an S-shape, and will appear to significantly extend the length of a sigmoid formed by the twisted field lines just under the arcade.

Another feature pertaining to the global three-dimensional CME structure can also be explained by the differential rotation of the field lines. Namely, a cavity region (Fig. 2a) forms because the field lines positioned initially just outside and just inside of the partition line are pulled apart by the differential motion of the footpoints thereby thinning the plasma. The three-part magnetic field lines configuration shown in Figure 2b,c,e,f suggests an interesting explanation for another observational result according to which roughly a third of all observed CMEs have the three-part (or concave-outward) features (St. Cyr O.C. et al. 2000). If we view the configuration from different angles while rotating it around z axis we observe that the three-part structure is visible for a total viewing angle of about 120°. This property of the configuration implies that there is an approximately 1/3 probability that randomly oriented CMEs would manifest a three-part structure.

It is known from observations that the pre-eruptive structures seen on the limb have the form of a helmet streamer (Hundhausen 1996), which is known to exist stably for long periods of time and can swell up for several days prior to eruption. Figure 4 shows the time evolution of the magnetic field lines that cross the z axis viewed in an xz plane as the photospheric velocity magnitude is increased from zero to a maximum value of the order of 0.01vA. The simulations show that if the increase of the magnitude of the velocity is stopped and the velocity is then maintained at a fixed level, the resulting configuration persists relatively unchanged (e.g. Figure 4c) for a long period of time. When acceleration is applied again the
field lines resume their increase in twist and the configuration continues to evolve. The rate of the dynamics seems to be closely related to the rate of change of the boundary velocity profile.

4.2. Structural changes induced by reconnection and the time profile of the photospheric velocity

If the evolution of the magnetic configuration proceeds to the point where the lower part of the arcade field lines approach the sigmoids (Fig. 3c), the finite resistivity in the system allows the arcade field lines to reconnect with the sigmoids. Figure 5a shows a structure which forms as a result of such a reconnection process. This configuration consists of part of the overlying arcade spliced to a short segment of the former sigmoid. Note that no unrealistically large boundary velocity is needed in order for this structure to form as a result of the reconnection. In this simulation the velocity magnitude at the boundary is driven from 0 to values of the order of 0.01$v_A$ over 300 Alfven times.

If we suppose that the sigmoidal field lines (orange field lines in Figure 2b,c) are in fact associated with the bright core of the CME which contains the prominence then we can provide an interesting interpretation to some observations pertaining to the behavior of the core during the eruption. Specifically, it has been observed that in some cases only a part of the embedded bright prominence gets pulled outward as a result of the eruption and part of it stays behind (Gilbert et al. 2000). If we look at the position of the sigmoids with respect to the arcades (Fig. 2b) we can see that the height of the reconnection point between the arcade and the sigmoids may vary depending on the exact spatial separation of the magnetic poles at the base plane and the spatial and temporal profile of the photospheric velocity. The footpoints of the arcade can converge close to the base-plane and in this case most of the conic shape formed by sigmoids (Fig. 2b) will be sliced off by the formation of the structure in Figure 5a. The reconnection might also be initiated higher up and then a part of the conic shape will stay behind. Note that the boundary velocity profile consisting of two co-rotating vortices is not a necessary condition for this process to take place. Only a converging flow that carries the footpoints of the overlying arcade towards the sigmoid is needed for the reconnection between these structures to take place and for the configuration shown in Figure 5a to form.

It is reasonable to expect that after the reconnection occurs the resulting structure (Fig.5a) is less stable than the configuration preceding the reconnection between the arcades and the sigmoids. While we plan to include more relevant physical processes into the model and address this question in detail in the future, the dynamics observed in the simulations
can help draw the following parallel with the observations. Recent observations (Sterling et al. 2000) of erupting CMEs propagating along the line of sight (halo CMEs) showed that the evolution proceeds from formation of an initial sigmoid to the development of an overlaying arcade positioned perpendicular to the sigmoid. Our model indicates that it takes more time for the larger, slower-moving yellow field lines to rotate around to form an arcade than for the low-lying orange field lines to form an S-shape; thus the sigmoid appears first and is then spanned and overshadowed by an arcade. Such rotation and twisting of the magnetic field lines a slow process. However, once the overlaying arcades come into close contact with the sigmoidal lines they can reconnect and if the resulting structure is unstable it can erupt quickly. Future simulations can demonstrate whether such scenario is a possible explanation of the eruptions.

For a maximum photospheric velocity magnitude of the order of 0.01vA the magnetic field lines twist by only one or a few turns before reconnection occurs and drastically changes the overall structure. Highly twisted magnetic field lines only form in numerical experiments where the photospheric velocity was ramped up to a magnitude of the order of 0.1vA and in this case the dynamics becomes very complicated with reconnection greatly reconfiguring the geometry. In such simulations the magnetic field would first undergo the dynamics described above but if we kept rapidly increasing the boundary velocity to a high magnitude the field lines became more twisted and due to complex dynamics the configuration seems to be much less connected to the imposed photospheric flow. Figures 5 b, c show some of the configurations formed as a result of this process. In contrast to the low velocity ramp-up, halting the ramp-up of the photospheric velocity in these high velocity cases does not yield magnetic configurations which persist relatively unchanged for long periods of time.

5. Conclusions and future work

The three-dimensional model presented in this paper suggests a possible scenario for the evolution of the coronal magnetic configurations. The numerical simulations showed that combined shearing and converging motions at the photosphere cause formation of several regions of geometrically distinct magnetic field lines. The nature of the dynamics that leads to the formation of such configurations seems to be robust with respect to the parameters in the model. The global configuration resulting from the formation of such regions bears a striking resemblance to characteristic features appearing in CME observations. In particular, when the three-dimensional magnetic structures are viewed from different angles projection effects account for the appearance of three-part structured eruptive configurations, sigmoids, overlaying arcades and heart-shaped features. We also found that if the
photospheric velocity magnitude is comparable with the observed speeds of the solar surface flows, reconnection-induced morphological changes can prevent the magnetic configurations from becoming highly twisted. Additionally, we proposed that the variability of the height of the reconnection point between different magnetic regions can explain splitting of the CME’s core prominence material in some eruptions. In general, the model seems to capture and unite several diverse observational results and explain them in terms of simple physical processes. Thus, our formulation of the problem appears to be promising and the following distinguishing features of the model should be particularly noted. Our numerical studies showed that exponential propagation methods offer significant advantages for MHD calculations by providing an accurate and efficient way to integrate MHD systems over long periods of time. In addition, we formulated boundary conditions which appear to give a good approximation to the realistic open domain dynamics in the corona.

In the future we will extend the model to include the effects of non-uniform density, gravity and finite $\beta$. In particular, we plan to investigate whether the density profile will reflect the dynamical changes in the magnetic field that we described in this paper. Since the model is formulated in a flexible way, it is possible to incorporate observational data into the initial and boundary conditions. With these extensions it would be possible to address more quantitative questions, perform direct comparisons with observations and conduct detailed studies of the rates of different physical processes throughout the evolution of the plasma configurations.

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REFERENCES


Fig. 1.— Initial and boundary conditions of the numerical simulations. (a) Field lines of the initial magnetic field generated by a magnetic dipole placed below the plane $z = 0$. Contour plot of the $z$-component of the magnetic field at the base plane. (b) Tangential velocity profile at the base plane $z = 0$. 
Fig. 2.— (a) A three-part (leading edge halo, cavity and bright core feature) CME observed by the SMM white-light coronagraph (reprinted from (Illing & Hundhausen 1983)). (b),(c) Different angle views of magnetic field lines forming a sigmoid and an overlaying arcade produced by the numerical simulation. (d) The same observed CME image with the enhanced brightness of the cavity region which reveals the heart shaped configuration. (e),(f) Magnetic field lines of a simulated CME including the lines that produce heart-shaped structure as a projection effect.
Fig. 3.— (a),(d) The magnetic field lines at the start of the rotation of their footpoints. Broken red line in (a) indicates a field line, or partition line, which connects centers of the two rotating vortices. (b),(e) The paths of the footpoints of these field lines in the base plane as the rotation proceeds. (c),(f) The geometry of the magnetic field lines shown in (a) and (d) once their footpoints have rotated according to the paths displayed in (b) and (e). In (c) the orange-like field lines form a sigmoid while the yellow-like field lines form an overlaying arcade. The green and purple field lines in (f) form a heart shaped feature when viewed as a projection shown in Figure 2e.
Fig. 4.— (a) Initial state of the magnetic field lines crossing the $z$ axis at different heights. (b), (c), (d) Time evolution of the magnetic field lines crossing $z$ axis viewed as a projection onto $xz$ plane as the photospheric velocity is ramped up to a maximum magnitude of the order of $0.01v_A$. The projection effects give the configuration an appearance of a swelling up helmet streamer.
Fig. 5.— (a) Magnetic lines configuration produced as a result of reconnection between the arcade-like (yellow line in Figure 3c) and sigmoid-like (orange line in Figure 3c) field lines. (b),(c) Twisted magnetic configurations formed when the photospheric velocity is ramped up to a maximum magnitude of the order of 0.1$v_A$. Structure in (c) is formed after the whole configuration expanded and multiple reconnection introduced various geometrical changes.